Geophys. J. R. astr. Soc. (1967) **14**, 287–295.

Viscosity of the Earth*

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Summary

Direct and indirect estimates of the variation of viscosity with depth in the mantle indicate that a low viscosity layer exists in the upper mantle. A viscosity varying with depth can be used to reconcile the various estimates of relaxation times. If the seismic anelasticity can be used as a guide the average viscosity of the lower mantle is about 10^{23} P. Combined with previous estimates of the upper mantle viscosity this gives a relaxation time of about 3000 years for the non-equilibrium bulge of the Earth. This is close to the time from the last ice age but is much less than the 10^7 years required if the non-equilibrium bulge is due to the changing rate of rotation which requires an average mantle viscosity of 10^{26} P. If the latter value is correct the activation volume for creep is much larger than for anelasticity or the effect of a phase change in the upper mantle is more effective in suppressing creep than attenuation.

The response of a layered viscous sphere to a surface load is calculated for a wide range of parameters including the above range of estimates for lower mantle viscosity. These results can be used to estimate the decay time, or the isostatic time scale, for various sized features.

Introduction

The long term rheological properties of the Earth are involved in mountain building, isostasy, geosynclinal subsidence and, in fact, most geological processes. These phenomena cannot conveniently be used to determine the rheological properties of the Earth since they are a function also of time varying stresses. The response of the Earth to loads placed on or removed from the surface on a time scale short compared to the ability of the Earth to respond and the response of the Earth to a known cyclic or steady body force are the kinds of experiments that can supply information about the long term non-elastic properties of the Earth. Examples of these processes are glacier loading and unloading, tidal response of the Earth to the attraction of the Moon and Sun, deformation due to changing rates of rotation and transient loads due to oceanic and atmospheric loading.

In spite of all these possibilities only the rates of uplift of Fennoscandia and Lake Bonneville and the non-equlibrium shape of the Earth have been used to estimate relaxation times. This 'direct' data has been supplemented by theoretical and semi-empirical estimates of the variation of viscosity with depth. There is increasing evidence that there is a zone of relatively low viscosity in the upper mantle.

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Depth variation

The variation of elastic properties with depth in the mantle is now fairly well known. The important features are the upper mantle low-velocity zone, the rapid increase in velocity in the transition region between about 300 and 800 km and a relatively gradual increase of velocity in the lower mantle. The low-velocity zone is consistent with the high thermal gradient in the upper mantle inferred from other considerations. The transition region probably contains two relatively sharp discontinuities which represent phase changes.

These discontinuities have been placed near 300 and near 600 kilometres. Self-compression controls the further increase of seismic velocities with depth in the lower mantle. The major discontinuities in the Earth, the crust–mantle interface and the mantle–core boundary are almost undoubtedly compositional in origin. The variation of viscosity with depth can be expected to show the same kind of structure. The high thermal gradient in the upper mantle can be expected to lower the viscosity; the collapse of silicates in the transition region can be expected to increase the viscosity rather abruptly, and the increasing importance of pressure can be expected to give an increasing viscosity in the lower mantle.

The estimates of viscosity by Haskell (1935, 1936), Crittenden (1963) and MacDonald (1963) were based on the assumption that the Earth was homogeneous.

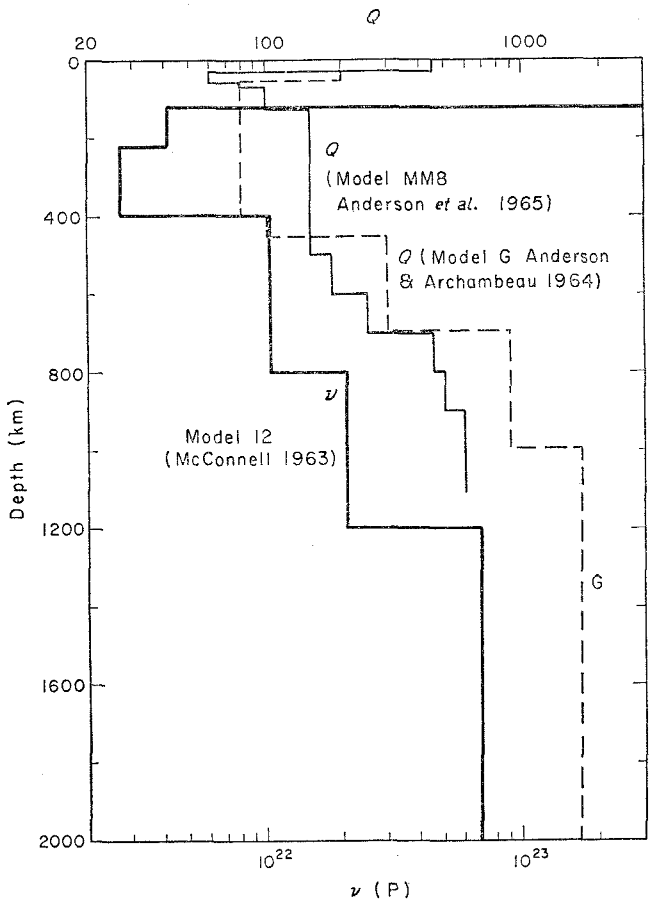

FIG. 1. Viscosity, ν, and shear wave anelasticity, Q, in the upper mantle.

Anderson (1966) and Takeuchi & Hasegawa (1965) showed that the discrepancies could be reconciled if the upper mantle had a lower viscosity than the lower mantle.

McConnell (1965) made the first experimental and theoretical study of the variation of viscosity with depth in the mantle. His preferred model based on Fennoscandia uplift data has a viscosity of about 3×10^{21} P in the upper mantle increasing to about 10^{22} P near 400 kilometres and 2×10^{22} P near 1000 kilometres. The general trend is remarkably similar to the seismic anelasticity as determined from surface wave and free oscillation experiments (Anderson & Archambeau 1964, Anderson, Ben-Menahem & Archambeau 1965). These results are compared in Fig. 1. This similarity was used by Anderson (1966) to estimate the viscosity on the lower mantle.

The non-equilibrium shape of the Earth and the phase lag of the solid Earth tides are potential sources of more direct data regarding the viscosity of the lower mantle.

It is now well established from satellite observations that the Earth is not in hydrostatic equilibrium. In particular the ellipticity, or oblateness, of the Earth is greater than would be appropriate for a liquid sphere rotating at the present rate. This has been attributed by MacDonald (1963) to the slowing down of the Earth's rate of rotation and by Wang (1966) to the melting of the polar icecaps. The former interpretation gives a relaxation time of the order of 10^7 years and an estimate of about 10^{26} P for the average viscosity of the mantle. The latter interpretation gives a relaxation time of about 5×10^3 years and an estimate of about 5×10^{22} P for the average viscosity of the mantle. Because of the non-symmetrical distribution of land and sea the melting of polar ice caps will also introduce departures from rotational symmetry, i.e. *pear-shaped* components. We do not yet have any estimates of the relative importance of these two possibilities. They must both contribute to some extent to the non-equlibrium shape of the Earth and there are also, of course, other possibilities.

Indirect estimates of viscosity

Theoretical estimates of the variation of viscosity with depth in the mantle have been made by Zharkov (1960), Cook (1963) and Gordon (1965). These calculations require estimates of such parameters as temperature, activation energy, activation volume and grain size as well as an assumption regarding mechanism. A low viscosity zone in the upper mantle is predicted for any reasonable estimate of the parameters. The dominant effect of temperature gives a viscosity decreasing with depth in the upper mantle; pressure dominates in the lower mantle and leads to an increasing viscosity with depth in this region of the Earth.

Anderson (1966) remarked that viscosity, or creep rate, and the seismic anelasticity are probably both defect controlled and would therefore be expected to be similar functions of depth through their temperature and pressure dependence.

High temperature creep and anelasticity measurements both depend on temperature according to $\exp E/RT$. For example, the creep rate of solids due to volume diffusion of vacancies yields an effective viscosity which can be written

$$\eta \sim \frac{kTl^2}{a^3 D}, \qquad D = D_0 e^{-E/RT},$$

where D is the coefficient of self-diffusion, E is the activation energy for self-diffusion, a is an atomic dimension and l is the grain size. This is the well-known *Nabarro-Herring equation.*

Similarly the anelasticity due to an alternating vacancy flux is (Friedel 1961)

$$Q^{-1} \sim \frac{e^{-E/RT}}{l^2 T}.$$

If these mechanisms are responsible for creep and seismic anelasticity, respectively, in the mantle and if both are controlled by the same activation energy, then

$$\eta/Q \sim \text{constant}.$$

Even if creep and anelasticity are not this closely related they are both probably controlled ultimately by diffusion processes and a close relationship is to be expected. Fig. 1 shows that such a relationship does indeed exist in the upper mantle. The ratio η/Q is roughly 4×10^{19} P. Kovach & Anderson (1964) measured average values of Q in shear for the whole mantle, the upper 600 km of the mantle and the lower 2300 km of the mantle. These values of Q are respectively 600, 200 and 2200, leading to estimates of average viscosity from equation of $2 \cdot 4 \times 10^{22}$ P for the whole mantle, 8×20^{21} for the upper mantle and 10^{23} for the lower mantle. The lowest Q measured seismically is about 60 in a thin layer at the base of the crust. This yields a value for viscosity of 2×10^{21} P. The lower mantle estimate is consistent with the non-equilibrium shape of the Earth if it is primarily due to the redistribution of ice and water since the last ice age. It is much lower than the estimate based on the assumption that it is due entirely to the decreasing rate of rotation. If the latter assumption is correct and the lower mantle has a viscosity as high as 10^{26} P then the mechanisms of creep and anelasticity are probably different. If they are both activated processes then the activation volume for creep is higher than for attenuation, i.e. the diffusion of the

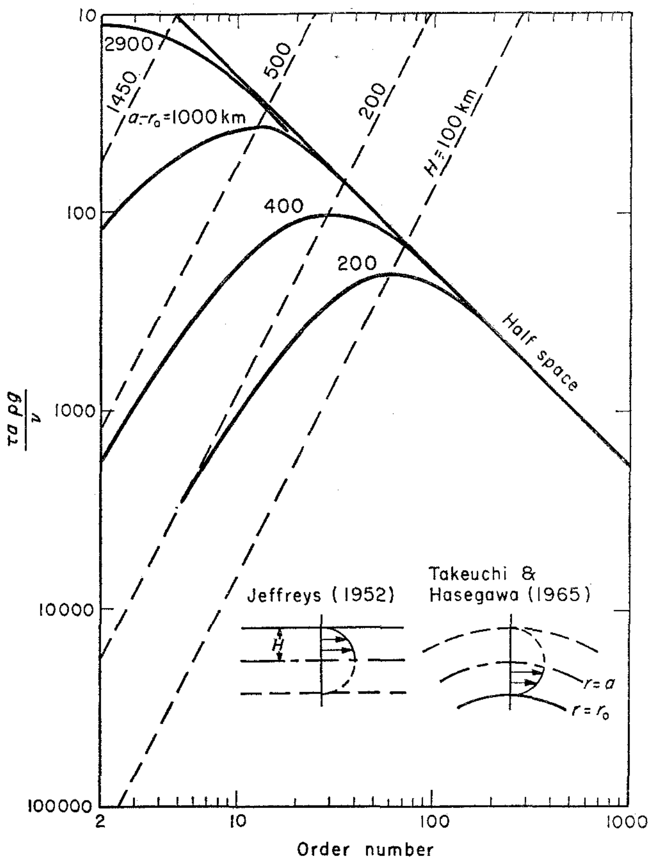

FIG. 2. Dimensionless relaxation time as a function of order number for viscous half-space, viscous layer and viscous spherical shell.

defects responsible for creep is suppressed more effectively by pressure than is the diffusion or reordering of the defects responsible for anelasticity. It is also possible, of course, that anelasticity is not an activated process and therefore depends only weakly on temperature and pressure. It is also an oversimplification to assume that a single mechanism is operative throughout the mantle and that the variation of properties depends only on temperature and pressure with unique values of the activation parameters. In particular the phase changes occurring between 300 and 800 km can very well predominate over the effects of temperature and pressure in this region. Due to the different crystal structure in the upper and lower mantle the activation energies and volumes in these two regions may be quite different. There is a general tendency for activation energy to increase with melting temperature and this alone would give different parameters for the phase assemblies in the upper and lower mantle.

Deformation of a layered viscous sphere

Present estimates of the Earth's viscosity have been based on models of the Earth taken as a homogeneous viscous half-space (Haskell 1935, 1936), a layered viscous half-space (McConnell 1965), a viscous layer over a rigid half-space (Jeffreys 1962)

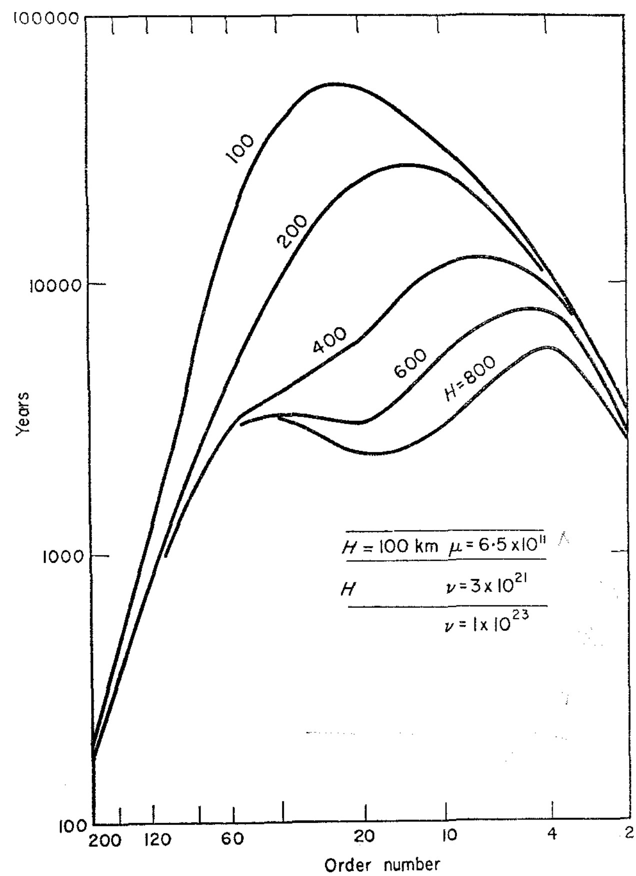

FIG. 3. Relaxation time for a layered viscous sphere with an elastic layer at the surface and inviscid liquid spherical core. The thickness of the first viscous layer is the parameter.

and a viscous spherical shell overlying a rigid sphere (Takeuchi & Hasegawa 1965). In all of these models the decay time, τ, is a function of the wavelength, λ, or order number, n, where

$$\lambda = \frac{2\pi a}{n+\frac{1}{2}}$$

is approximately the wavelength for a sphere of radius a. Fig. 2 summarizes this relationship for these models. The models with a rigid lower mantle predict a minimum relaxation time for wavelengths of the order of 400 to 4000 km depending on the thickness of the flowing layer. For a viscous layer 400 km thick the relaxation time decreases by an order of magnitude as n decreases from 2 to 30. Features having wavelengths of the order of half the Earth's circumference therefore will persist longer than features with a wavelength of the order of 1000 km. A more realistic calculation must take into account the viscosity of the lower mantle.

The response of a layered viscous sphere to surface loading originally formulated by Takeuchi & Hasegawa (1965), has been generalized and programmed for an IBM 7094 computer. The results presented here are for an elastic crust overlying a two-layered viscous mantle overlying an inviscid fluid core. The relaxation time as a function of wavelength (order number) is computed for various combinations of viscosities and thicknesses of the mantle layers. The thickness of the rigid 'crustal' layer is taken as 100 km in all cases. The range of viscosities considered bracket most of those that have been proposed for the various regions of the mantle. The order

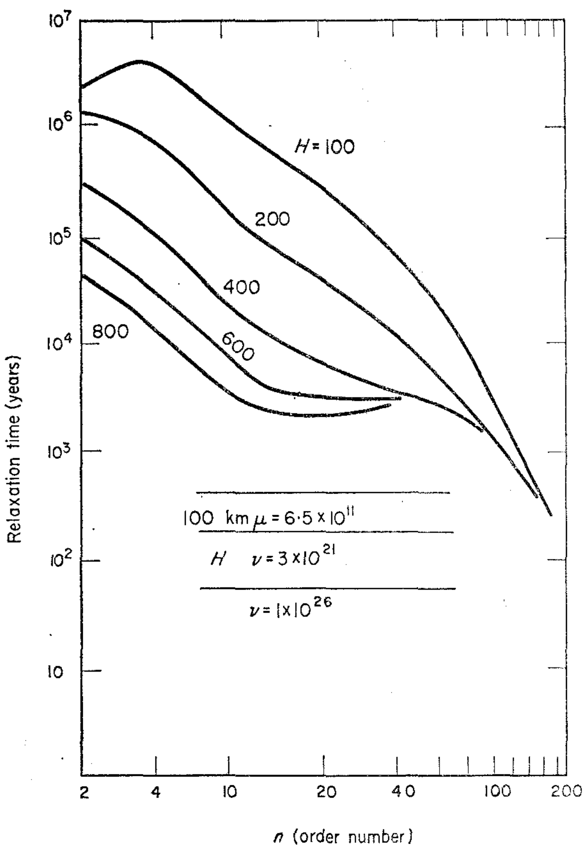

FIG. 4. Same as Fig. 3 for a higher viscosity lower mantle.

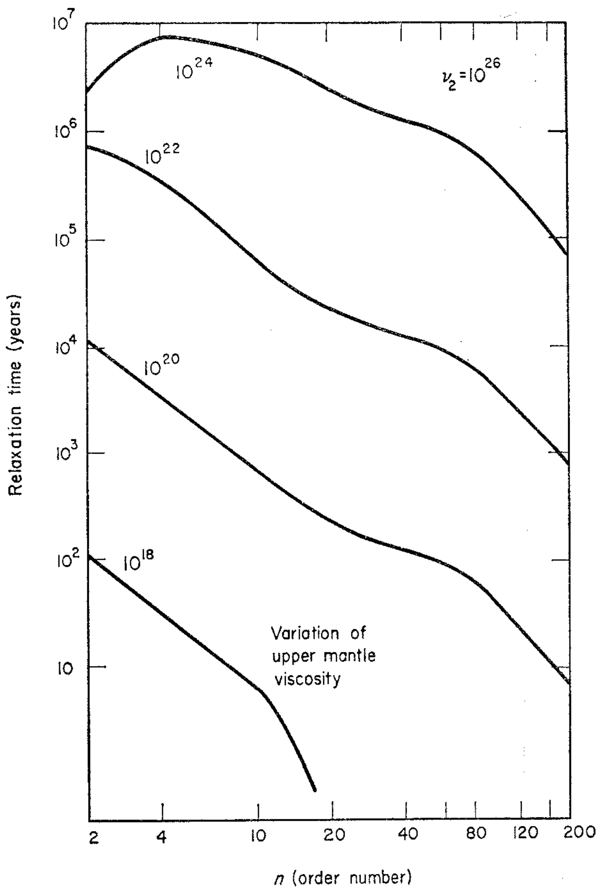

FIG. 5. Effect of upper mantle viscosity on relaxation time of a layered viscous sphere.

numbers to be associated with the Earth's bulge, the dimensions of Fennoscandia and the dimensions of the Lake Bonneville uplift are, respectively, 2, 20–60 and 200. Relaxation times associated with these features have been estimated as 5×10^3 to 10^7, 4×10^3 to 10^3, and 10^3 years respectively.

The first case considered (Fig. 3) is for an upper mantle with a viscosity of 3×10^{21} P overlying a lower mantle with a viscosity of 10^{23} P. The thickness of the upper mantle is the variable. The relaxation time for the Earth's bulge considered as a surface load, is about 3×10^3 years and is roughly independent of the thickness of the low viscosity upper mantle. The relaxation time reaches a maximum between $n = 4$ and 30 depending on the upper mantle thickness. Intermediate wavelengths will persist longer than the very large or very small features.

In Fig. 4 the lower mantle viscosity is 10^{26} P. This model gives relaxation times that decrease with decreasing wavelength. The relaxation time for $n = 200$ is roughly 170 years for these two cases.

Fig. 5 gives results for various upper mantle viscosities for an upper mantle 400 km thick and a lower mantle viscosity of 10^{26} P. Fig. 6 shows the effect of varying the lower mantle viscosity. A relaxation time of about 10^6 years is the maximum that can be achieved for this kind of model, regardless of the lower mantle viscosity.

These calculations represent preliminary results of a more detailed study to be published later.

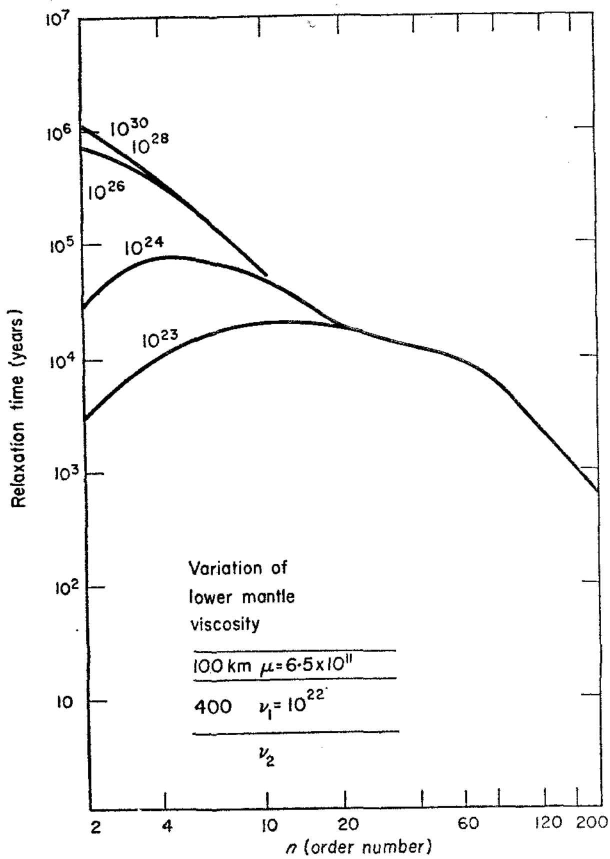

FIG. 6. Effect of lower mantle viscosity on relaxation time of a layered viscous sphere.

Acknowledgments

This work was supported by the Sloan Foundation and we gratefully acknowledge their assistance.

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1966 *June*.

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Discussion

The idea of a direct relationship between the apparent viscosity of the mantle and the attenuation of seismic waves is not supported by either solid state theory or by laboratory observations. When a polycrystalline solid is cooled from high temperature in the laboratory successive damping peaks will be observed while the strength continually increases. Large dāmping peaks are even found at temperatures near absolute zero where the effective viscosity of the material is nearly infinite. The basic reason for this behaviour is that damping may result from the stress-induced motion of highly mobile atom or defect species while the 'viscosity' or plastic response to a continuously applied stress depends on the mobility of the slowest moving atom species making up the crystal.

The very real possibility that the high damping of the upper mantle may result from the stress-induced flow of partially melted material through intergranular spaces also rules out any reliable correlation between the seismic attenuation in the upper mantle and the viscosity of the lower mantle.

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